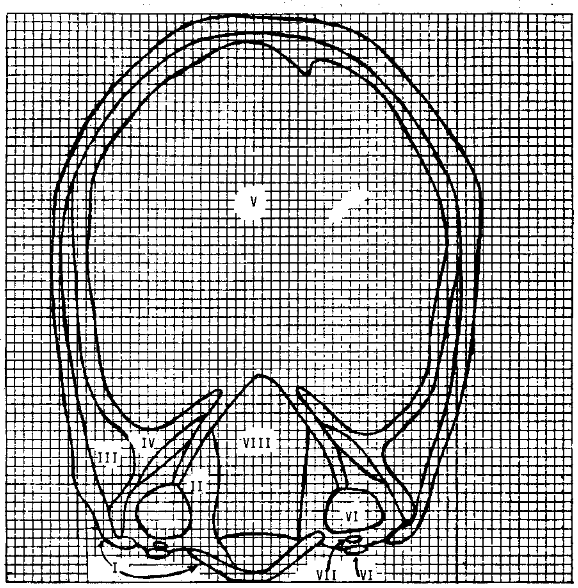

TISSUE	ϵ_r	σ
I. SKIN	44.5	1.01
II. FAT	6.4	0.12
III. MUSCLE	50.5	1.3
IV. BONE	6.4	0.12
V. BRAIN	48.0	1.25
VI. HUMOR	80.0	1.9
VII. LENS	50.5	1.3
VIII. NASAL	25	.6

64 x 64 = 4096 CELLS
FREQUENCY = 1 GHz
LENGTH = 20 cm
# NONZERO CELLS = 2600

Fig. 1. An eight-tissue model of a human cross section through the eyes. The dielectric values used were taken from [6].

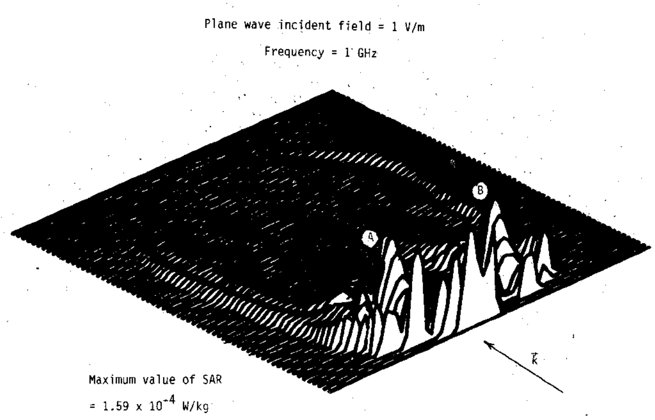

Fig. 2. Isometric plot of the FFT-calculated SAR distribution for the head cross section.

humor of the eyes. These peaks may be due to the fact that the size of the vitreous humor bodies are very close to one internal wavelength at this frequency.

The very shallow deposition at frequencies about 1 GHz may suggest the possibility of inhomogeneous modeling of only 2 to 3 skin depths into the body, reducing the number of unknowns drastically.

V. Conclusions

This paper illustrates the ability of the FFT method to obtain high-resolution SAR distributions for the two-dimensional TM absorption problem. Because the more general three-dimensional electric-field integral equation is also a convolution, it should be possible to extend the approach to the three-dimensional absorption problem.

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Characteristic Impedance of the Slab Line with an Anisotropic Dielectric

HISASHI SHIBATA, YUKIO KIKUCHI, AND RYUITI TERAKADO

***Abstract*—The characteristic impedance of the slab line with a circular conductor having an anisotropic dielectric is presented by using the affine and conformal transformations. Moreover, a simpler approximate formula of the impedance expressed in terms of $\epsilon_{\|}$, $\epsilon_{\perp}$, and r/h is also presented, where $\epsilon_{\|}$, $\epsilon_{\perp}$, r, and h are the principal axes-relative dielectric constants of the anisotropic dielectric, the inner conductor radius, and the half length between ground planes, respectively.**

I. Introduction

The slab line with a concentric circular conductor having an isotropic dielectric between parallel ground planes is used as the slotted section in microwave measurements and has been analyzed by many authors [1]–[5].

Due to the stability of its electrical properties, an anisotropic dielectric has been used in microwave integrated circuits. For various shielded striplines and covered microstrips, the analyses and some of the effects resulting from the utilization of an anisotropic dielectric have been reported [6]–[16].

This paper presents the characteristic impedance of the slab line with an anisotropic dielectric shown in Fig. 1. The permittivity tensor of the anisotropic dielectric in Fig. 1 is presented as

$$\bar{\epsilon}(x,y)=\begin{bmatrix}\epsilon_{\|} & 0\\ 0 & \epsilon_{\perp}\end{bmatrix}. \tag{1}$$

The application of the structure of Fig. 1 is not extensive. However, it is useful to analyze the line of Fig. 1 because it is a more general case than just including an isotropic dielectric.

It is useful to apply a transform method [6], [10] for the analysis of the slab line shown in Fig. 1. By the affine transfor-

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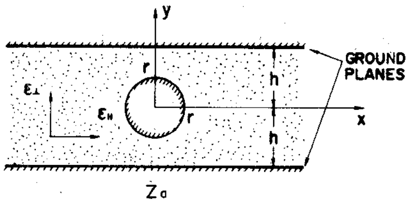

Fig. 1. Cross section of the slab line with a circular conductor having an anisotropic dielectric (Z_a: characteristic impedance).

mation, the shape of the inner conductor is transformed into an ellipse where either the major axis or the minor axis is parallel with the ground planes. Then, the Das and Rao's analysis [19], [20] can be applied to the equivalent isotropic slab line. Thus, the characteristic impedance of the slab line shown in Fig. 1 is presented. The impedance data will be presented for sapphire, i.e., a single crystal of aluminum oxide, and for pyrolytic boron nitride. An approximate formula relating the characteristic impedance to $\epsilon_{\|}$, $\epsilon_{\perp}$, and r/h is also presented by using a property of the elliptic integral.

II. Characteristic Impedance

In this section, the characteristic impedance of the slab line with an anisotropic dielectric shown in Fig. 1 is presented. First, we apply an affine transformation [13, eq. (3)] to the region of Fig. 1. In Fig. 1, the angle θ between the principal axes of the anisotropic dielectric and the ground planes is zero. Therefore, β and α in the transformation are zero and $\sqrt{\epsilon_{\|}/\epsilon_{\perp}}$, respectively.

Using the transformation, the region of Fig. 1 is transformed into a region with an isotropic dielectric of which the permittivity equals $\epsilon_0\sqrt{\epsilon_{\|}\epsilon_{\perp}}$. In the equivalent isotropic slab line, the width between the ground planes becomes $2\alpha h$ and the shape of the inner conductor is transformed into an ellipse with the major and minor axes being r and αr, respectively.

Such a slab line with the elliptical conductor has been analyzed by Das and Rao [19], [20]. Strictly, their analysis, which uses the Schwartz–Christoffel transformation, is to be used for an oval conductor. However, this analysis is a good approximation for the elliptical conductor and can be applied to the equivalent isotropic slab line.

The characteristic impedance Z_a of the slab line shown in Fig. 1 is obtained as

$$Z_a = \frac{30\pi}{\sqrt[4]{\epsilon_{\|}\epsilon_{\perp}}} \cdot \frac{K'(k)}{K(k)} \tag{2a}$$

where k is related to h and r by the expression

$$\frac{h}{r} = \pi \left\{ \frac{1}{\sqrt{\frac{\epsilon_{\|}}{\epsilon_{\perp}}} \cdot \ln \frac{1+k}{1-k}} + \frac{1}{2 \sin^{-1} k} \right\}. \tag{2b}$$

In (2), $K(k)$, $K'(k)$, and k are complete elliptic integrals of the first kind and the argument of the elliptic integrals, respectively.

Fig. 2 shows Z_α versus r/h for sapphire ($\epsilon_{\|}=11.6, \epsilon_{\perp}=9.4$) [17] and pyrolytic boron nitride ($\epsilon_{\|}=5.12, \epsilon_{\perp}=3.4$) [18]. From Fig. 2, it is obvious that the dielectric anisotropy has a weak effect for a small wire, but the characteristic impedance for a large wire is influenced considerably by the direction of the principal axes of the anisotropic dielectric.

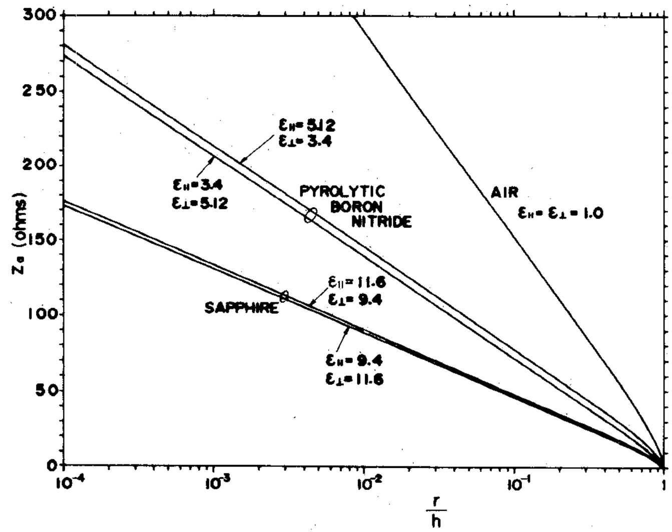

Fig. 2. Characteristic impedances versus r/h.

TABLE I
Comparison of the Characteristic Impedances Z_a using (2) with those using (3)

	Z_a (ohms)			
	P.B.N. $\epsilon_{\|}=5.12$ $\epsilon_{\perp}=3.4$		SAPPHIRE $\epsilon_{\|}=11.6$ $\epsilon_{\perp}=9.4$	
$\frac{r}{h}$	Eq. (2)	Eq. (3)	Eq. (2)	Eq. (3)
0.7	17.919	20.425	10.667	12.058
.6	23.496	24.953	14.152	14.921
.5	29.477	30.309	17.895	18.306
.4	36.413	36.864	22.243	22.449
.3	45.093	45.314	27.698	27.791
.2	57.136	57.224	35.285	35.319
.1	77.564	77.585	48.182	48.189

When we take $\epsilon_{\|}=\epsilon_{\perp}$ in (2), the formula is directly applicable to the slab line with an isotropic dielectric. An example of the application is added in Fig. 2 when the medium is air.

Z_a and r/h in (2) are related with each other through the argument k of the elliptic integral. Another approximate formula relating Z_a with $\epsilon_{\|}$, $\epsilon_{\perp}$, and r/h is presented here by using an approximate relation between K'/K and k [21]. If it is possible to put a restriction on the range of Z_a, then a simpler approximate formula can be obtained in the following form:

$$Z_a = \frac{60}{\sqrt[4]{\epsilon_{\|}\epsilon_{\perp}}} \ln\left\{ \frac{h}{r} \Big/ \frac{\pi}{8}\left(\sqrt{\frac{\epsilon_{\perp}}{\epsilon_{\|}}}+1\right)\right\} \quad \text{(for small wire)}. \tag{3}$$

Equation (3) is obtained by using (21) in [21, pp. 22] and the following relation:

$$\ln \frac{1+k}{1-k} \cong 2k, \qquad \sin^{-1}k \cong k \qquad \text{(for small } k\text{)}.$$

The comparison between the characteristic impedance by (2) and that by (3) is shown in Table I.

III. Conclusion

We have presented the characteristic impedance of the slab line with an anisotropic dielectric. The characteristic impedance has been obtained analytically by using transform methods. A simpler approximate formula which is useful for application has also been presented.

Acknowledgment

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Edge-Guided Magnetostatic Mode in a Ridged-Type Waveguide

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Abstract — A ridged-type magnetostatic waveguide is analyzed using the boundary element method. A bias magnetic field is applied perpendicularly to the surface of an yttrium-iron-garnet (YIG) film grown on a gadlinium-gallium-garnet (GGG) substrate. The dispersion curves and the potential profiles obtained in this paper show that the mode has a strong nonreciprocal property and is a kind of edge-guided mode which propagates along either side of the ridge, depending upon the direction of the bias field and the direction of the wave propagation. In addition, the authors emphasize the fact that the boundary element method is useful for analysis of a complex structure in the field of magnetostatic wave (MSW) devices.

I. Introduction

In a previous paper [1], the authors have already shown that the boundary element method (BEM) [2] is very effective and useful for the analysis of magnetostatic wave (MSW) problems. In the present paper, a ridged-type waveguide will be treated. Tanaka and Shimizu [3] obtained the dispersion relation for the same type of waveguide as discussed here, but the bias magnetic field was applied in the plane of the yttrium-iron-garnet (YIG) film and, therefore, the mode properties obtained there are quite different from those revealed here. Moreover, they used the equivalent-circuit method to get the results and, hence, did not show any potential profile.

For the purpose of application of MSW to microwave integrated circuits, it is desirable that a bias magnetic field be applied in the normal direction to the YIG film grown on the gadlinium-gallium-garnet (GGG) substrate. As is well known, however, only a magnetostatic volume wave (MSVW) can propagate in an infinite and homogenous YIG film.

Now, notice that the ridged structure has a side parallel or tilted to the bias field, and we might expect that the side or the wall of the ridge can support a kind of magnetostatic surface wave (MSSW). We may suggest that this type of guided wave stems from almost the same idea as guided MSW's in the plate of YIG magnetized nonuniformly [4]–[6]. Thus, it is very interesting to investigate the characteristics of the wave propagation along the ridged structure, and, besides, the authors would like to emphasize the fact that the BEM is very suitable for the analysis of a complex structure like this one.

II. BEM Formulation

The BEM approach for MSW propagation problems is described briefly below. A waveguide to be considered is shown in Fig. 1. A cross section of a waveguide may be arbitrary, but an internal dc magnetic field is supposed to be uniform for the sake of mathematical simplicity. Under a quasistatic approximation,

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